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FREE CONVECTION IN A POROUS MEDIUM WITH MAGNETIC FIELD, VARIABLE PHYSICAL PROPERTIES AND VARYING WALL TEMPERATURE

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Abstract
Effects of magnetic field, variable viscosity and variable thermal conductivity on similarity solutions of free convection at a vertical flat plate in a porous medium are studied numerically. Excess of plate temperature over the temperature of the ambient fluid is assumed to vary as a power of function of x, where x is the distance measured along the plate. The flow and heat transfer quantities are found to be functions of C, λ, γ_μ, γ_k where C is magnetic interaction parameter, λ is power of index of the plate temperature and γ_μ, γ_k are viscosity variation coefficient and thermal conductivity variation coefficient respectively. The cases of hot plate and cold plate are considered. Interesting features of the solutions are presented and discussed.

Key Words: Free Convection; Variable Physical properties; Magnetic field; varying wall temperature
2000 Mathematics Subject Classification Codes: 76S05; 76R10; 76DXX

NOMENCLATURE

B_0 = Magnetic flux
C - Magnetic interaction parameter
f - Dimensionless stream function
g - Acceleration due to gravity
h - Local heat transfer coefficient
K Permeability
K^* - Porous parameter, $\frac{L^2}{K}$
k – Thermal conductivity of the saturated porous medium
M^2 - Hartmann number, $\frac{B_0^2 L^2 \sigma}{\mu_f}$
p – Pressure
Ra_x – *Rayleigh number*, $\frac{\rho_\infty K g \beta |T_o - T_\infty| x}{\mu_f \alpha_m}$
T_0 - Temperature of plate
T_∞ - Ambient temperature
T_f - Reference temperature
u, v - Velocity components in x- and y- directions
x, y – Cartesian coordinates

Greek Symbols

α_m - Effective thermal diffusivity of the porous medium
β - Coefficient of thermal expansion
γ_μ - Viscosity variation coefficient
γ_k - Thermal conductivity variation coefficient
η - Similarity variable
θ - Dimensionless temperature
μ - Dynamic viscosity
ρ - Fluid density
ψ - Stream function
λ – Power of index of plate temperature

Subscripts

0 – Condition at the plate
∞ - Condition at infinity
f - Condition at reference temperature

I.INTRODUCTION

Heat transfer studies in porous media find applications in several Engineering and technological systems, as pointed out in [4]. In view of the applications, several heat transfer studies were carried out and a few are quoted here. Reference [6] and reference [3] have discussed free convection in a porous medium adjacent to vertical and horizontal surfaces when the surface temperature varies as a power function of distance. Reference [5] studied the effect of magnetic field and varying plate temperature on convective heat transfer past a vertical plate in porous medium.

Reference [1] discussed the effect of variable viscosity on convective heat transfer in three different cases of natural convection, mixed convection and forced convection, taking fluid viscosity to vary inversely with temperature.

In the present paper the effects of magnetic field, variable viscosity, variable thermal conductivity and varying plate temperature on free convection at a vertical plate in a porous medium are studied. The excess of the plate temperature over the ambient is assumed to vary as power function of distance x along the plate, viscosity and thermal conductivity are assumed to vary as linear functions of temperature and a magnetic field is assumed to act normal to the plate. Similarity solutions are obtained for the problem when the plate temperature is higher as well as lower than the ambient temperature.

II.FORMULATION AND SOLUTION

Let a flat plate be embedded vertically in a porous medium saturated with a viscous incompressible homogeneous quiescent fluid (see figure 1 for the physical model and the coordinate system). The porous medium is assumed to be homogeneous and is in thermal equilibrium with the surrounding fluid. Let a magnetic field of uniform strength be applied in a direction normal to the plate. Let *X*-axis be taken along the plate and *Y*-axis perpendicular to it. The temperature of the plate (T_0) is assumed to vary as a power function of distance along the plate as, $T_0 = T_\infty + A x^\lambda$, where T_∞ is temperature of the ambient fluid, *A* is a constant and λ is a real number. Viscosity and thermal conductivity are assumed to be functions of temperature as, $\mu = \mu_f s_\mu(t)$, $k = k_f\, s_k(t)$, where μ_f, k_f are viscosity and thermal conductivity evaluated at the film temperature,

$$s_\mu(t) = 1 + \left(\frac{d\mu}{dt}\right)_f \left(T - T_f\right),$$

$$s_k(t) = 1 + \left(\frac{dk}{dt}\right)_f (T - T_f),$$

and $T_f \left(= \dfrac{T_0 + T_\infty}{2} \right)$ is the film temperature.

A viscosity variation coefficient γ_μ and a thermal conductivity variation coefficient γ_k are introduced as

$$\gamma_\mu = \frac{1}{\mu_f}\left(\frac{d\mu}{dT}\right)_f \left(T_0 - T_\infty\right) \text{ and}$$

$$\gamma_k = \frac{1}{k_f}\left(\frac{dk}{dt}\right)_f (T_0 - T_\infty).$$

Density of the fluid is assumed to be a function of temperature only in the body force term and the other fluid properties are assumed to be constant. The boundary layer equations governing the Darcy model are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

$$\frac{\partial p}{\partial x} + \rho g + \sigma B_0^2 u + \frac{\mu}{K} u = 0$$

(2)

$$\frac{\partial p}{\partial y} + \frac{\mu}{K} v = 0$$

(3)

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y} \right)$$

(4)

where u, v are fluid velocity components, *T* is fluid temperature, K is Permeability, k is thermal conductivity of the fluid saturated porous medium, B_0 is the magnetic flux and other symbols have their usual meanings. Taking, $\rho = \rho_\infty [1 - \beta(T - T_\infty)]$, in the body force term in (2), introducing a stream function ψ and eliminating fluid pressure from (2) and (3), the governing equations are obtained as

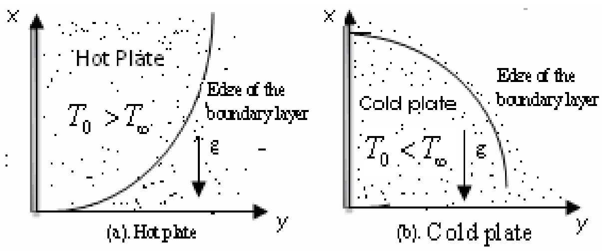

Fig-1. Physical model and coordinate system

$$\left(\mu + K\sigma B_0^2\right)\left(\frac{\partial^2 \psi}{\partial y^2}\right) + \left(\frac{\partial \psi}{\partial y}\right)\left(\frac{\partial \mu}{\partial y}\right) = K g \rho_\infty \beta \left(\frac{\partial T}{\partial y}\right)$$

(5)

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\left(k \frac{\partial^2 T}{\partial y^2} + \frac{\partial k}{\partial y}\frac{\partial T}{\partial y} \right)$$

(6)

The boundary conditions on *T* and ψ are

$$\left.\begin{aligned} &at \quad y = 0,\ T = T_0,\ \frac{\partial \psi}{\partial x} = 0, \\ &as \quad y \to \infty,\ T \to T_\infty,\ \frac{\partial \psi}{\partial y} \to 0 \end{aligned}\right\}$$

(7)

Introducing *Rayleigh number* (Ra_x), Hartman number (M^2), a magnetic interaction parameter *C* and non dimensional functions f, θ together with a similarity variable η through the relations

$$\left.\begin{aligned}
Ra_x &= \frac{\rho_\infty K\, g\beta\left|T_0 - T_\infty\right| x}{\alpha_m \mu_f} \\
M^2 &= \frac{B_0^2 L^2 \sigma}{\mu_f} \\
K^* &= \frac{L^2}{K} \\
C &= \frac{K^*}{K^* + M^2} \\
f(\eta) &= \frac{\psi}{\alpha Ra_x^{\frac{1}{2}}} \\
\theta(\eta) &= \frac{T - T_\infty}{T_0 - T_\infty} \\
\eta &= \frac{y}{x} Ra_x^{\frac{1}{2}}
\end{aligned}\right\}$$

(5), (6) are rewritten as

$$\left[1 + C\gamma_\mu\left(\theta - \frac{1}{2}\right)\right] f'' + C\gamma_\mu f'\theta' = C\theta'$$

(9)

$$[1+\gamma_k(\theta - 1/2)]\,\theta'' + \gamma_k\,\theta'^2 - \lambda f'\theta + \frac{(1+\lambda)}{2} f\theta' = 0$$

(10)

The boundary conditions (7) become

$$\left.\begin{aligned}
&at\ \ \eta = 0, \quad \theta = 1, \quad f = 0, \\
&as\ \eta \to \infty, \ \theta \to 0, \ f' \to 0
\end{aligned}\right\}$$

(11)

Equation (9) can be integrated once using the condition on f' at infinity to get

$$f' = \frac{C\theta}{\left[C\gamma_\mu\left(\theta - \frac{1}{2}\right) + 1\right]}$$

(12)

Evaluating this expression at $\eta = 0$ we get the slip velocity $f'(0)$ as

$$f'(0) = \frac{2C}{C\gamma_\mu + 2}\ .$$

For constant viscosity (i.e., $\gamma_\mu = 0$), the slip velocity becomes equal to C.

III.a PARAMETERS OF THE PROBLEM AND THEIR EFFECT ON THE FLOW AND HEAT TRANSFER

The flow and heat transfer quantities are functions of the parameters, *C*, the magnetic interaction parameter (parameter depending on the porosity of the medium and Hartmann number), γ_μ, the viscosity variation coefficient, γ_k, the thermal conductivity variation coefficient and λ, the power of index of plate temperature.

When the constant *A* appearing in the expression for the temperature of the plate takes positive or negative values, the temperature of the plate becomes higher or lower than the ambient temperature. In the former case we can say that plate is hot and in the later case that the plate is cold.

The parameters γ_μ and γ_k can take positive as well as negative values, the limiting values for both being '+2' and '-2'. For liquids (except for water near 4^o C and for uid metals) negative values of γ_μ and positive values of γ_k correspond to $T_0 > T_\infty$ (hot plate) while positive values of γ_μ and negative values of γ_k to $T_0 < T_\infty$ (cold plate). However for gases it is vice versa. Zero values for γ_μ and γ_k indicate that variations of viscosity and thermal conductivity with temperature are neglected and they do not mean that T_0 and T_∞ are equal. In this paper, solutions are found for the values -1, 0, 1 of γ_μ and γ_k.

The parameter *C* takes smaller values (less than unity) when either the porous parameter takes smaller values or the Hartmann number takes larger values, that is, when porosity of the medium is high or the intensity of the magnetic field is high. When there is no applied magnetic field M^2 takes zero value and as a result *C* takes the value unity. Solutions are found for the values 0.1, 0.5 and 1 of *C*. Reduced flow can be expected for smaller values of *C* or for increased intensity of the magnetic field as the magnetic field lines obstruct the flow.

To determine certain important values for λ, the total heat convected in the flow, $Q(x)$ at any down stream location x is considered.

$$Q(x) = \int_0^\infty \rho\, C_p \beta\left(T - T_\infty\right) u\, dy\ .$$

This can be seen to be proportional to $x^{\frac{3\lambda+1}{2}}$. For uniform heat flux surface, $Q(x)$ should vary linearly with x and so $\lambda = \frac{1}{3}$. For an adiabatic surface, $Q(x)$ should be independent of x and so $\lambda = \frac{-1}{3}$. Zero value of λ corresponds to the isothermal case. In this study solutions are found for the values -0.3, -0.2, 0.0, 0.3, 0.5 and 1.0 of λ. For positive values of the constant *A*, an increase in the value of λ can correspond to an increase in the temperature of the plate, and, in a broader sense, it can result in enhanced flow.

The effects of simultaneous variation of the values of the parameters on the flow and heat transfer are presented in the discussion.

III.b NUMERICAL SOLUTION

The equations for f and θ i.e., (10), (12) are integrated numerically subject to appropriate boundary conditions by Runge-Kutta-Gill method [2], together with a shooting technique. Results of the present analysis when *C*=1, $\lambda = 0, \gamma_\mu = 0, \gamma_k = 0$ (no magnetic field, isothermal plate, constant fluid properties) agree well with appropriate results of [1]. Also, results of our analysis when $\gamma_\mu = 0, \gamma_k = 0$ (constant fluid properties) agree with corresponding results of [5].

IV. DISCUSSION OF THE RESULTS

Interesting results related to the quantities of practical utility, namely the shear stress, heat transfer coefficient, velocity and temperature are presented, certain numerical results in the form of tables I,II and some other results in the form of figures 2-8. Quantities like the local Nusselt number and the local drag coefficient can readily be obtained from the heat transfer coefficient and the skin friction.

Table- II
Thermal boundary layer thickness for
$\gamma_\mu = 1.0 \;\&\; \gamma_k = -1.0$

C \ λ	-0.3	0	0.5	1
1	29	27	24	22
0.5	42	40	35	32
0.1	91	86	76	68

Table- I
Thermal boundary layer thickness for
$\lambda = 0.5$ & $\gamma_K = 0$

γ_μ \ C	1.0	0.5	0.1
-1	18	26	61
0	19	27	58
1	19	26	54

From tables I, II we may notice that thermal boundary layer thickness diminishes as λ as well as *C* take increasing values. However variation in boundary layer thickness with changing values of γ_μ is not quite significant.

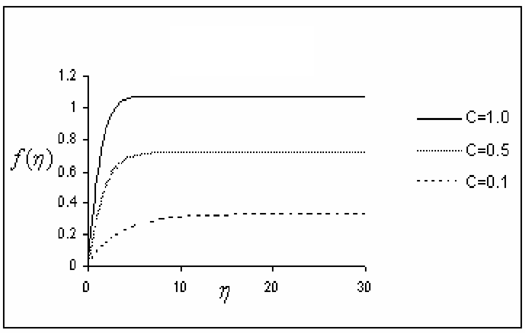

Fig-4 Stream function for $\gamma_\mu = 1$ & $\lambda = 1.0$

Plots of slip velocity and heat transfer coefficient are presented in figures 2 and 3. The slip velocity as well as the heat transfer coefficient take larger values for $T_0 > T_\infty$ $(\gamma_\mu < 0, \gamma_k > 0)$ than for $T_0 < T_\infty$ $(\gamma_\mu > 0, \gamma_k < 0)$ (for liquids), this behaviour being more pronounced in the absence of magnetic field than in its presence. They also diminish with increasing intensity of the magnetic field (i.e., as *C* diminishes) and with diminishing values of λ. For all values of *C*, the heat transfer coefficient increases with increasing values of λ. Plots of the stream function for different values of *C* are shown in figure 4. Stream function takes diminishing values with increasing intensity of the magnetic field and also with increasing values of λ. Variations in shear stress with γ_k are shown in figure 5. Shear stress is observed to take negative values and also its absolute value at the plate increases with diminishing values of γ_k.

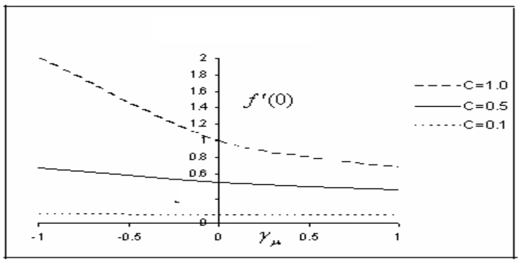

Fig - 2 Slip velocity, $f'(0)$ for $\lambda = 0.5$

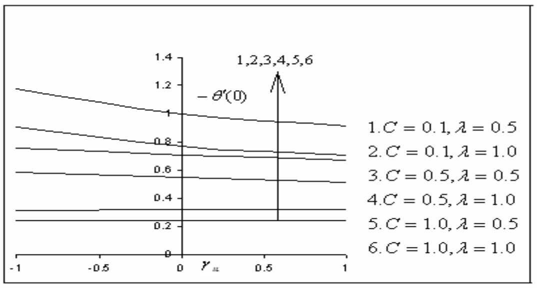

Fig-3 - Variation of '$-\theta'(0)$' With γ_μ for $\gamma_k = 0$

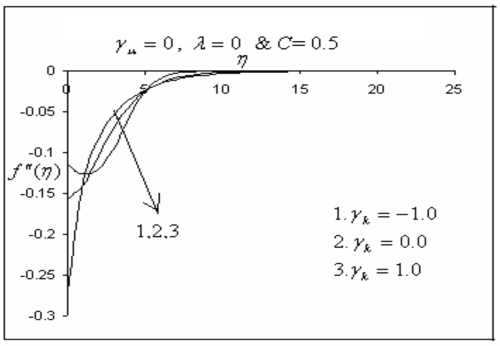

Fig-5 Variation of shear stress with η for $\gamma_\mu = 0$, $\lambda = 0$, $C = 0.5$

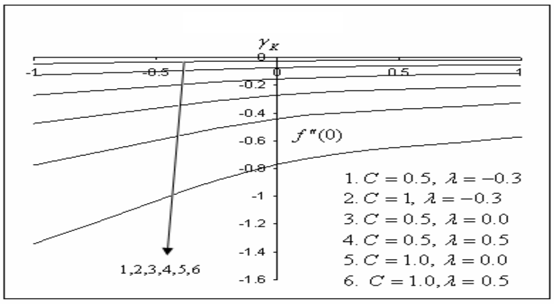

Fig-6 Variation of **Skin friction** with γ_K **for** $\gamma_\mu = 0$

Variation of skin friction with γ_k for different values of C and λ are shown in figure 6. Absolute values of skin friction can be observed to increase with increasing values C and λ. Absolute value of skin friction also assumes relatively larger values for negative values of γ_k, smaller values for γ_k =0 and much smaller values for positive values of γ_k. Plots of non dimensional velocity $u\,(i.e., f'(\eta))$ are shown in figure 7. For liquids, the velocity at the plate takes larger values for $T_0 > T_\infty$ ($\gamma_\mu = -1, \gamma_k = 1$), smaller values when variations in μ and k are neglected ($\gamma_\mu = 0,\ \gamma_k = 0$), and much smaller values for $T_0 < T_\infty$ ($\gamma_\mu = 1, \gamma_k = -1$). Hydrodynamic

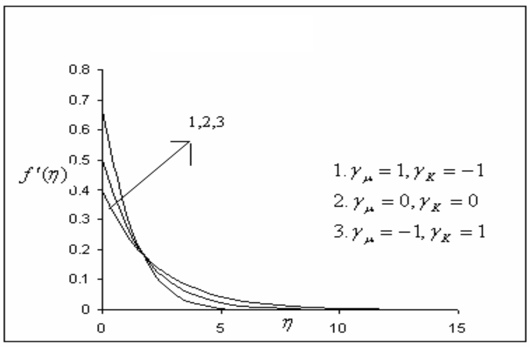

Fig -7 Non dimensional velocity $f'(\eta)$ **for** $C = 0.5 \& \lambda = 0.5$

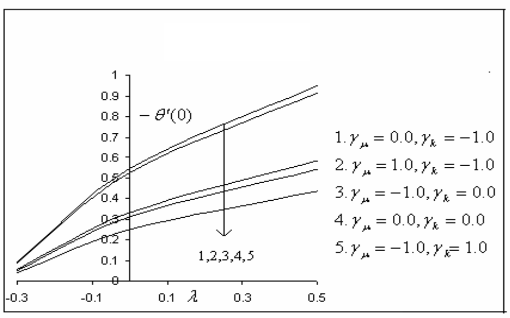

Fig-8 Variation of **Heat transfer coefficient** with λ **for** C=0.5

boundary layer thickness can be observed to be more for $T_0 < T_\infty$ than for $T_0 > T_\infty$. Variations in heat transfer coefficient with λ are shown in figure 8. The heat transfer coefficient can be seen to take increasing values with increasing values of λ and with diminishing values of γ_k, for all values of γ_μ. Heat transfer coefficient is also observed to take increasing values as γ_μ takes diminishing values.

V.CONCLUSIONS:

Skin friction $f''(0)$ takes negative values for all values of parameters. Heat transfer coefficient '$-\theta'(0)$' takes larger values with diminishing values of *C* and with increasing values of λ. Thermal boundary layer thickness increases with diminishing values of *C,* γ_k (for a fixed value of the porous parameter) and this indicates that, in a particular porous medium the transfer of heat is governed by the intensity of the magnetic field. Hydrodynamic boundary layer thickness is more when the plate is cold than when the plate is hot.

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